① Solve the system

$$\begin{cases} 3x + 2y - z = 4 \\ x + y + 3z = 1 \\ -2x + y - 5z = -5 \end{cases}$$

② Multiply

ⓐ $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ⓑ $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 3 \\ -2 & 1 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

③ Solve $\frac{1}{3} \cdot 3x = 151$ without using division

guess & check

$X = \frac{1}{3} \cdot 151$

$X = 50\frac{1}{3}$

$\frac{2}{3} \longrightarrow \frac{3}{2}$

$3 \longrightarrow \frac{1}{3}$

$\frac{13}{4} \longrightarrow \frac{4}{13}$

① Solve the system

$$\begin{cases} 3x+2y-z=4 \\ x+y+3z=1 \quad (-3) \\ -2x+y-5z=-5 \end{cases}$$

$3x+2y-z=4$

$-3x-3y-9z=-3$

$-y-10z=1$

$2x+2y+6z=2$

$-2x+y-5z=-5$

$3y+z=-3$

(3) $-y-10z=1$

$3y+z=-3$

$-3y-30z=3$

$3y+z=-3$

$-29z=0$

$z=0$

$3y+0=-3$

$y=-1$

$x+y+3z=1$

$x+-1+0=1$

$x=2$

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\cdot1+4\cdot0 & 2\cdot0+4\cdot1 \\ 1\cdot1+3\cdot0 & 1\cdot0+3\cdot1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

Identity Matrix
= the "1" for multiplication

$7 \cdot 1 = 7$

$13 \cdot 1 = 13$

$57.28 \cdot 1 = 57.28$

$$\begin{bmatrix} 3 & 7 & 2 \\ 1 & 4 & 5 \\ 8 & 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 7 & 2 \\ 1 & 4 & 5 \\ 8 & 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 7 \\ 3 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2 \times \underline{3} \qquad \underline{3} \times 3$

(b) $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 3 \\ -2 & 1 & -5 \end{bmatrix} \begin{matrix} 3 \\ \cdot z \\ -1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \cdot x + 2 \cdot y + -1 \cdot z \\ 1 \cdot x + 1 \cdot y + 3 \cdot z \\ -2x + 1 \cdot y - 5 \cdot z \end{bmatrix}$

$= \begin{matrix} 3x + 2y - z \\ 1x + 1y + 3z \\ -2x + y - 5z \end{matrix}$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \cdot X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A \cdot X = B$ can't divide w/ matrices

so we need to multiply by reciprocal

$3 \cdot \frac{1}{3} = 1$

$\frac{2}{3} \cdot \frac{3}{2} = 1$

$\frac{4}{13} \cdot \frac{13}{4} = 1$

$A \cdot A^{-1} = I$

A: matrix

A^{-1}: reciprocal

I: identity matrix

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$4 \cdot a \quad 2 \cdot b$

$3 \cdot c \quad 1 \cdot d$

$2a + 1c = 1$

$4a + 3c = 0$

$2b + 1d = 0$

$4b + 3d = 1$

$3 \cdot \frac{1}{3} = 1$

$7 \cdot \frac{1}{7} = 1$

$A \cdot A^{-1} = 1$

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

original matrix

$$A^{-1} = \begin{bmatrix} 1.5 & -0.5 \\ -2 & 1 \end{bmatrix}$$

Inverse matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

identity matrix

$A \cdot X = B$

$X = A^{-1} \cdot B$

Find the inverse of each matrix

(a) $A = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

$A \cdot A^{-1} = \underline{I}$

(b) $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

B^{-1} = No Solution

B is a singular matrix

(a) $A = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix}$

(b) $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

determinant $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

$\det A = 2 \cdot 4 - 4 \cdot 1 = \boxed{4}$

$\det B = 2 \cdot 2 - 4 \cdot 1 = \boxed{0}$ → Singular matrix

The inverse of a 2x2 matrix

$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A = \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{4} \begin{bmatrix} 4 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

Find the inverse

$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$\det A = 2 \cdot 3 - 4 \cdot 1 = \boxed{2}$

$A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1.5 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix}$

A^{-1}

- Turn in classwork
- Turn in 4.1–4.3
- HW

  Sect. 4.3 #17, 18

  Sect. 4.5 #1, 2, 14, 15, 18

→ Sect. 4.1 #1–12
Sect. 4.2 #3–17 (odd)
Sect. 4.3 #1, 4, 9, 11–16, 20–24